Torque

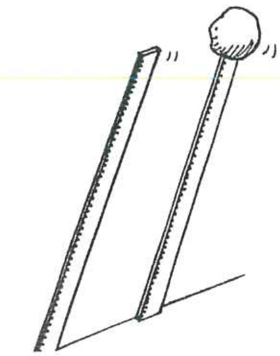

FIGURE 8.15 Which stick has the greater rotational inertia about its bottom end? When released, which will rotate to the floor first?

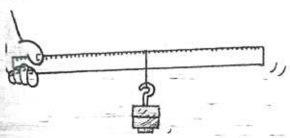

FIGURE 8.16 Move the weight away from the hand and you feel the difference between force and torque.

Hold the end of a meterstick horizontally with your hand. Dangle a weight from it near your hand and you can feel the stick twist. Now slide the weight farther from your hand and the twist is more. But the weight is the same. The force acting on your hand is the same. What's different is the *torque*.

A torque (rhymes with *dork*) is the rotational counterpart of force. Force tends to change the motion of things; torque tends to twist or change the state of rotation of things. If you want to make a stationary object move, apply force. If you want to make a stationary object rotate, apply torque.

Torque differs from force just as rotational inertia and regular inertia differ: both torque and rotational inertia involve distance from the axis of rotation. In the case of torque, this distance, which provides leverage, is called the **lever arm.** It is the shortest distance between the applied force and the rotational axis. We define **torque** as the product of this lever arm and the force that tends to produce rotation:

$$\text{Torque} = \text{lever arm} \times \text{force}$$

Torques are intuitively familiar to youngsters playing on a seesaw. Kids can balance a seesaw even when their weights are unequal. Weight alone doesn't produce

FIGURE 8.17 Since ancient times, mass has been measured by balancing torques.

CHECK YOUR ANSWERS

1. Stand the hammer with the handle at your fingertip and the head at the top. Why? Because it will have more rotational inertia this way and be more resistant to a rotational change. Those acrobats you see on stage who balance their friends at the top of a long pole have an easier task when their friends are at the top of the pole. A pole empty at the top has less rotational inertia and is more difficult to balance!
2. Try it and see! (If you don't have clay, fashion something equivalent.)
3. Not so fast on this one. Give it some thought if you haven't come up with an answer. Then look to the end of the chapter for an answer.

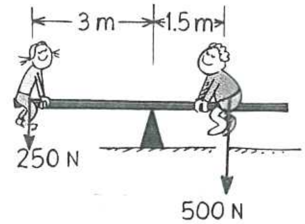

FIGURE 8.18 No rotation is produced when the torques balance each other.

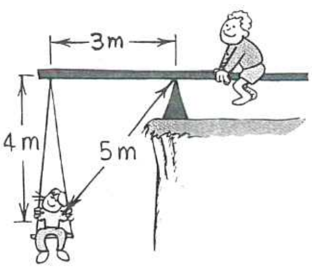

FIGURE 8.19 The lever arm is still 3 m.

rotation. Torque does, and children soon learn that the distance they sit from the pivot point is every bit as important as weight (Figure 8.18). The torque produced by the boy on the right tends to produce clockwise rotation, while torque produced by the girl on the left tends to produce counterclockwise rotation. If the torques are equal, making the net torque zero, no rotation is produced.

Recall the equilibrium rule in Chapter 2—that the sum of the forces acting on a body or any system must equal zero for mechanical equilibrium. That is, $\Sigma F = 0$. We now see an additional condition. The *net torque* on a body or on a system must also be zero for mechanical equilibrium. Anything in mechanical equilibrium doesn't accelerate—neither translationally nor rotationally.

Suppose that the seesaw is arranged so that the half-as-heavy girl is suspended from a 4-meter rope hanging from her end of the seesaw (Figure 8.19). She is now 5 meters from the fulcrum, and the seesaw is still balanced. We see that the lever-arm distance is still 3 meters and not 5 meters. The lever arm about any axis of rotation is the perpendicular distance from the axis to the line along which the force acts. This will always be the shortest distance between the axis of rotation and the line along which the force acts.

This is why the stubborn bolt shown in Figure 8.20 is more likely to turn when the applied force is perpendicular to the handle, rather than at an oblique angle as shown in the first figure. In the first figure the lever arm is shown by the dashed line and is less than the length of the wrench handle. In the second figure, the lever arm is equal to the length of the wrench handle. In the third figure the lever arm is extended with a pipe to provide more leverage and a greater torque.

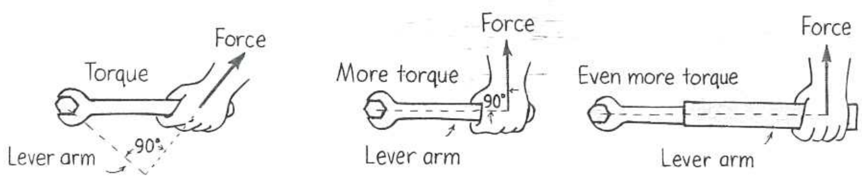

FIGURE 8.20 Although the magnitudes of the force are the same in each case, the torques are different.

CHECK YOURSELF

1. If a pipe effectively extends a wrench handle to three times its length, by how much will the torque increase for the same applied force?
2. Consider the balanced seesaw in Figure 8.18. Suppose the girl on the left suddenly gains 50 N, such as by being handed a bag of apples. Where should she sit in order to balance, assuming the heavier boy does not move?
3. How do these principles apply to the position of a doorknob on a conventional door?

CHECK YOUR ANSWERS

1. Three times more leverage for the same force gives three times more torque. (This method of increasing torque sometimes results in shearing off the bolt!)
2. She should sit $\frac{1}{2}$ m closer to the center. Then her lever arm is 2.5 m. This checks: 300 N × 2.5 m = 500 N × 1.5 m.
3. The doorknob is placed far from the hinges, thus making a longer lever arm. A push or pull on the knob should be perpendicular to the door. The perpendicular component of force opens it, whereas any component parallel to the door helps to rip the hinges off the wall.

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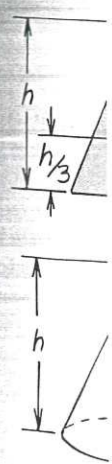

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Center of Mass and Center of Gravity

Throw a baseball into the air, and it will follow a smooth parabolic trajectory. Throw a baseball bat spinning into the air, and its path is not smooth; its motion is wobbly, and it seems to wobble all over the place. But, in fact, it wobbles about a very special place, a point called the **center of mass** (CM).

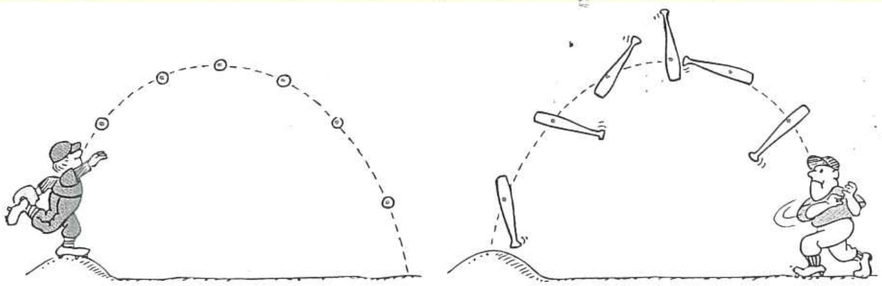
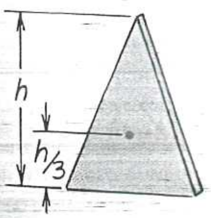


FIGURE 8.21 The center of mass of the baseball and that of the bat follow parabolic trajectories.

For a given body, the center of mass is the average position of all the mass that makes up the object. For example, a symmetrical object such as a ball has its center of mass at its geometrical center; by contrast, an irregularly shaped body such as a baseball bat has more of its mass toward one end. The center of mass of a baseball bat therefore is toward the thicker end. A solid cone has its center of mass exactly one-fourth of the way up from its base.

Center of gravity (CG) is a term popularly used to express center of mass. The center of gravity is simply the average position of weight distribution. Since weight and mass are proportional, center of gravity and center of mass refer to the same point of an object.* The physicist prefers to use the term *center of mass,* for an object has a center of mass whether or not it is under the influence of gravity. However, we shall use either term to express this concept and favor the term *center of gravity* when weight is part of the picture.



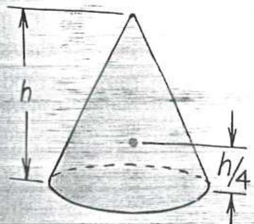


FIGURE 8.22 The center of mass for each object is shown by the red dot.

The multiple-flash photograph (Figure 8.23) shows a top view of a wrench sliding across a smooth horizontal surface. Note that its center of mass, indicated by the white dot, follows a straight-line path, while other parts of the wrench wobble as they move across the surface. Since there is no external force acting on the wrench, its center of mass moves equal distances in equal time intervals. The motion of the spinning wrench is the combination of the straight-line motion of its center of mass and the rotational motion about its center of mass.

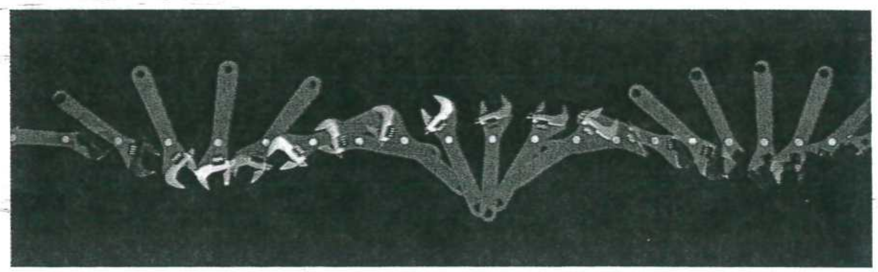


FIGURE 8.23 The center of mass of the spinning wrench follows a straight-line path.

*For almost all objects on and near the Earth, these terms are interchangeable. There can be a small difference between center of gravity and center of mass when an object is large enough for gravity to vary from one part to another. For example, the center of gravity of the World Trade Center is about 1 millimeter below its center of mass. This is due to the lower stories being pulled a little more strongly by Earth's gravity than the upper stories. For everyday objects (including tall buildings!) we can use the terms *center of gravity* and *center of mass* interchangeably.

If the wrench were instead tossed into the air, no matter how it rotates its center of mass (or center of gravity) would follow a smooth parabola. The same is true for an exploding cannonball (Figure 8.24). The internal forces that act in the explosion do not change the center of gravity of the projectile. Interestingly enough, if air resistance can be neglected, the center of gravity of the dispersed fragments as they fly though the air will be in the same place the center of gravity would be if the explosion didn't occur.

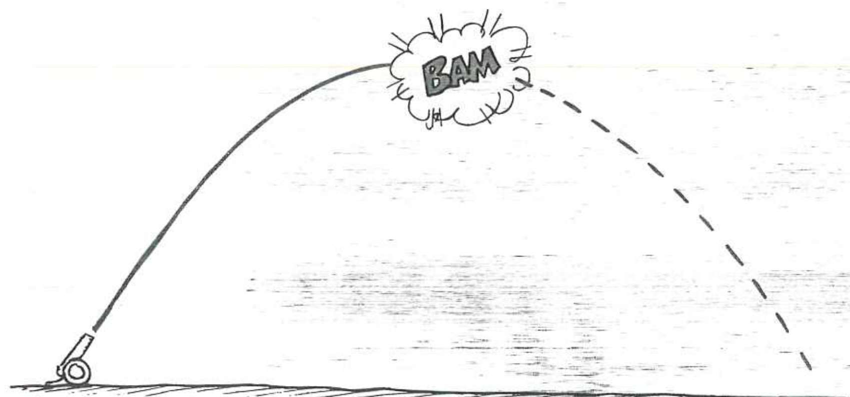


FIGURE 8.24 The center of mass of the cannonball and its fragments move along the same path before and after the explosion.

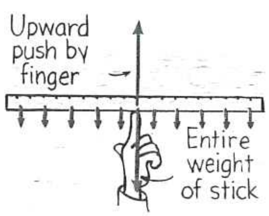


FIGURE 8.25 The weight of the entire stick behaves as if it were concentrated at its center.

Locating the Center of Gravity

The center of gravity of a uniform object such as a meter stick is at its midpoint, for the stick acts as though its entire weight were concentrated there. Supporting that single point supports the whole stick. Balancing an object provides a simple method of locating its center of gravity. In Figure 8.25 the many small arrows represent the pull of gravity all along the meter stick. All these can be combined into a resultant force acting through the center of gravity. The entire weight of the stick may be thought of as acting at this single point. Hence, we can balance the stick by applying a single upward force in a direction that passes through this point.

The center of gravity of any freely suspended object lies directly beneath (or at) the point of suspension (Figure 8.26). If a vertical line is drawn through the point of suspension, the center of gravity lies somewhere along that line. To determine exactly where it lies along the line, we have only to suspend the object from some other point and draw a second vertical line through that point of suspension. The center of gravity lies where the two lines intersect.

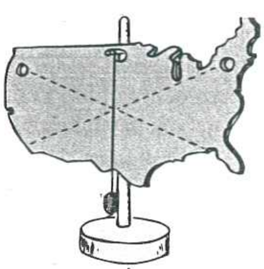


FIGURE 8.26 Finding the center of gravity for an irregularly shaped object.

The center of mass of an object may be a point where no mass exists. For example, the center of mass of a ring or a hollow sphere is at the geometrical center where no matter exists. Similarly, the center of mass of a boomerang is outside the physical structure, not within the material making up the boomerang (Figure 8.27).

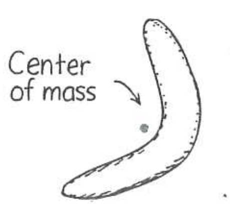


FIGURE 8.27 The center of mass can be outside the mass of a body.

Stability

The location of the center of mass is important for stability (Figure 8.29). If we drop a line straight down from the center of mass of an object of any shape and it falls inside the base of the object, it is in stable *equilibrium;* it will balance. If it falls

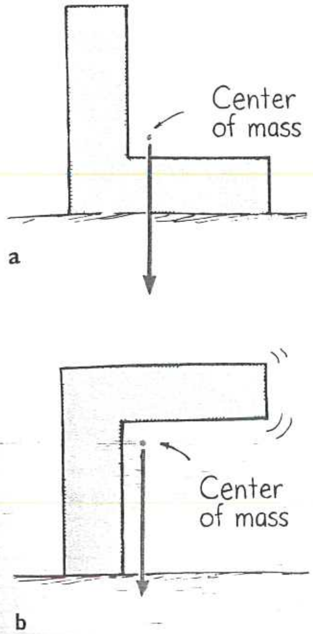


FIGURE 8.29 The center of mass of the L-shaped object is located where no mass exists. In (*a*), the center of mass is above the base of support, so the object is stable. In (*b*), it is not above the base of support, so the object is unstable and will topple over.

FIGURE 8.28 The athlete executes a "Fosbury flop" to clear the bar while her center of gravity passes beneath the bar.

outside the base, it is unstable. Why doesn't the famous Leaning Tower of Pisa topple over? As we can see in Figure 8.30, a line from the center of gravity of the tower falls inside its base, so the Leaning Tower has stood for several centuries.

To reduce the likelihood of tipping, it is usually advisable to design objects with a wide base and low center of gravity. The wider the base, the higher the center of gravity must be raised before the object will tip over.

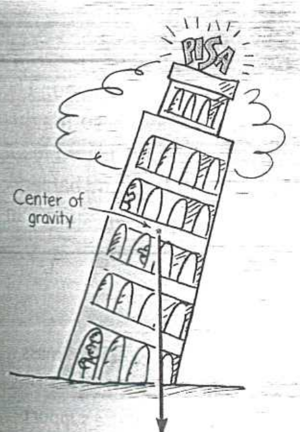


FIGURE 8.30 The center of gravity of the Leaning Tower of Pisa lies above the base of support, so the tower is in stable equilibrium.

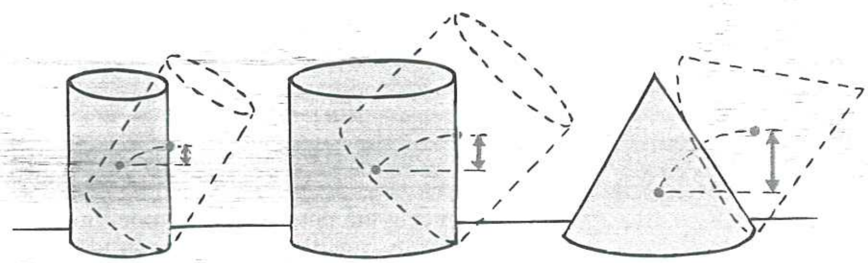


FIGURE 8.31 The vertical distance that the center of gravity is raised in tipping determines stability. An object with a wide base and low center of gravity is more stable.

When you stand erect (or lie flat), your center of gravity is within your body. Why is the center of gravity lower in an average woman than in an average man of the same height? Is your center of gravity always at the same point in your body? Is it always inside your body? What happens to it when you bend over?

If you are fairly flexible, you can bend over and touch your toes without bending your knees—providing you are not standing with your back against a wall. Ordinarily, when you bend over and touch your toes, you extend your lower extremity as shown in Figure 8.33 left, so that your center of gravity is above a base of support, your feet. If you attempt to do this when standing against a wall, however, you cannot counterbalance yourself, and your center of gravity soon protrudes beyond your feet as at the right in Figure 8.33. You are off-balance and you rotate.

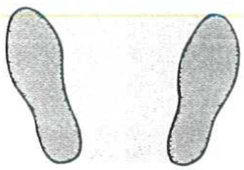


FIGURE 8.32 When you stand, your center of gravity is somewhere above the area bounded by your feet. Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?

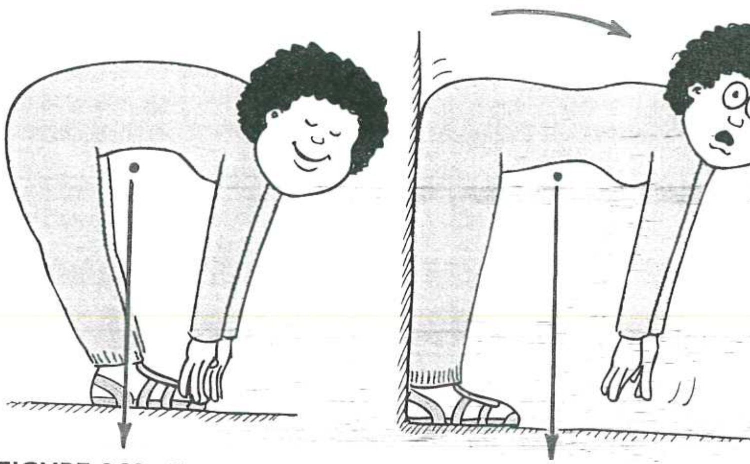


FIGURE 8.33 You can lean over and touch your toes without falling over only if your center of gravity is above the area bounded by your feet.

You rotate because of an unbalanced torque. This is evident in the two L-shaped objects shown in Figure 8.34. Both are unstable and will topple unless fastened to the level surface. It is easy to see that even if both shapes have the same weight, the one on the right is more unstable. This is because of the greater lever arm and, hence, greater torque.

CHECK YOURSELF

1. Where is the center of mass of the Earth's atmosphere?
2. Why is it dangerous to slide open the top drawers of a fully loaded file cabinet that is not secured to the floor?
3. When a car drives off a cliff, why does it rotate forward as it falls?

Try balancing the pole end of a broom upright on the palm of your hand. The support base is quite small and relatively far beneath the center of gravity, so it's difficult to maintain balance for very long. After some practice you can do it if you learn to make slight movements of your hand to exactly respond to variations in balance. You learn to avoid under-responding or over-responding to the slightest variations in

CHECK YOUR ANSWERS

1. Like a giant basketball, the Earth's atmosphere is a spherical shell with its center of mass at the Earth's center.
2. The filing cabinet is in danger of tipping because the CG may extend beyond the support base. Then torque due to gravity tips the cabinet.
3. When all wheels are on the ground, the car's CG is above a support base. But when the car drives off a cliff, the front wheels are first to leave the ground and the support base shrinks to the line between the rear wheels. So the car's CG extends beyond the support base and it rotates, as would the Leaning Tower of Pisa if its CG extended beyond its support base.

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