## Chapter 4 Density and Buoyancy

Will it float or will it sink? If you are designing ships this is a very important question. The largest ship in the world is the Jahre Viking, an oil-carrying tanker. This super-sized ship is 1,504 feet long and 264 feet wide, longer than 5 football fields laid end-to-end. If the Empire State building was laid on its side, the Jahre Viking would be longer by 253 feet! Crew members use bicycles to get from place to place on the ship. The ship is too large to fit through the Panama or Suez Canal, and it cannot dock in many of the world's seaports. The Jahre Viking is largely constructed of steel - so how can a big, heavy ship like this actually float? By the time you finish studying this chapter on density and buoyancy, you will be able to explain how ships and boats of all shapes and sizes can float.

## Key Questions

1. What is density and how can you measure it?
2. What two things does density depend on?
3. How does a steel ship float, when a steel marble sinks?


### 4.1 Density

When you think about the many kinds of matter you come into contact with every day, what properties come to mind? Some matter is solid and hard, like steel and wood. Some matter is liquid like water, or a gas like air. And in the category of solid matter, there are big differences. A block of wood and a block of steel may be the same size but one has a lot more mass than the other. Because of that difference in mass, wood floats in water and steel sinks. Whether an object floats or sinks in water is related to its density. This chapter will explain density, a property of all matter.

## Density is a property of matter

Density is mass Density describes how much mass is in a given volume of a material. per unit volume Steel has high density; it contains 7.8 grams of mass per cubic centimeter. Aluminum, as you well might predict, has a lower density; a one-centimeter cube has a mass of only 2.7 grams.


The density of
Liquids and gases are matter and have density. The density of water water and air is about one gram per cubic centimeter. The density of air is lower, of course - much lower. The air in your classroom has a density of about 0.001 grams per cubic centimeter.
density - the mass of matter per unit volume; density is typically expressed in units of grams per milliliter ( $\mathrm{g} / \mathrm{mL}$ ), grams per cubic centimeter ( $\mathrm{g} / \mathrm{cm}^{3}$ ), or kilograms per cubic meter $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$.

## Comparative densities

(vary with temperature and pressure)


Figure 4.1: The density of steel, aluminum, water, and air expressed in different units.

## Units of density

Density in units Your laboratory investigations will typically use density in units of of grams per grams per milliliter ( $\mathrm{g} / \mathrm{mL}$ ). The density of water is one gram per milliliter milliliter. That means one milliliter of water has a mass of one gram, and 100 milliliters of water a mass of 100 grams.

Density in $\mathbf{g} / \mathbf{c m}^{\mathbf{3}}$ Some problems use density in units of grams per cubic centimeter and $\mathbf{k g} / \mathbf{m}^{\mathbf{3}} \quad\left(\mathrm{g} / \mathrm{cm}^{3}\right)$. Since one milliliter is exactly the same volume as one cubic centimeter, the units of $\mathrm{g} / \mathrm{cm}^{3}$ and $\mathrm{g} / \mathrm{mL}$ are actually the same. For applications using large objects, it is more convenient to use density in units of kilograms per cubic meter ( $\mathrm{kg} / \mathrm{m}^{3}$ ). Table 4.1 gives the densities of some common substances in both units.
Converting units of density

To convert from one to the other, remember that $1 \mathrm{~g} / \mathrm{cm}^{3}$ is equal to $1000 \mathrm{~kg} / \mathrm{m}^{3}$. To go from $\mathrm{g} / \mathrm{cm}^{3}$ to $\mathrm{kg} / \mathrm{m}^{3}$ you multiply by 1,000 . For example, the density of ice is $0.92 \mathrm{~g} / \mathrm{cm}^{3}$. This is the same as $920 \mathrm{~kg} / \mathrm{m}^{3}$.
To go from $\mathrm{kg} / \mathrm{m}^{3}$ to $\mathrm{g} / \mathrm{cm}^{3}$ you divide by 1,000 . For example, the density of aluminum is $2,700 \mathrm{~kg} / \mathrm{m}^{3}$. Dividing by 1,000 gives a density of $2.7 \mathrm{~g} / \mathrm{cm}^{3}$.


Convert between units of density

A reference book lists the density of ceramic tile as $2,650 \mathrm{~kg} / \mathrm{m}^{3}$. Estimate the mass of one cubic centimeter of tile.
$\begin{array}{ll}\text { 1. Looking for: } & \text { Mass of } 1 \mathrm{~cm}^{3} \text {, which is the density in } \mathrm{g} / \mathrm{cm}^{3} \\ \text { 2. Given: } & \text { Density of } 2,650 \mathrm{~kg} / \mathrm{m}^{3} \\ \text { 3. Relationships: } & 1 \mathrm{~g} / \mathrm{cm}^{3}=1,000 \mathrm{~kg} / \mathrm{m}^{3} \\ \text { 4. Solution: } & \begin{array}{l}\text { Divide by } 1,000 \text { to get the density in } \mathrm{g} / \mathrm{cm}^{3} . \\ \end{array} \\ 2,650 \div 1,000=2.65 \mathrm{~g}\end{array}$

## Your turn...

a. A bronze statue has a density of $6,000 \mathrm{~kg} / \mathrm{m}^{3}$. What is the density in $\mathrm{g} / \mathrm{mL}$ ? Answer: $6 \mathrm{~g} / \mathrm{mL}$

Table 4.1: Densities of common substances

| Material | $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :---: | :---: |
| Platinum | 21,500 | 21.5 |
| Lead | 11,300 | 11.3 |
| Steel | 7,800 | 7.8 |
| Titanium | 4,500 | 4.5 |
| Aluminum | 2,700 | 2.7 |
| Glass | 2,700 | 2.7 |
| Granite | 2,600 | 2.6 |
| Concrete | 2,300 | 2.3 |
| Plastic | 2,000 | 2.0 |
| Rubber | 1,200 | 1.2 |
| Liquid water | 1,000 | 1.0 |
| Ice | 920 | 0.92 |
| Oak (wood) | 600 | 0.60 |
| Pine (wood) | 440 | 0.44 |
| Cork | 120 | 0.12 |
| Air (avg.) | 0.9 | 0.0009 |

Figure 4.2: The densities of some common materials.


Figure 4.3: The density range of common materials.

## Solving problems

A four-step The method for solving problems has four steps. Follow these steps
technique and you will be able to see a way to the answer most of the time and will at least make progress toward the answer almost every time. There is often more than one way to solve a problem. Sometimes you will have to use creativity to work with the given information to find the answer. Figure 4.4 shows the steps for solving problems.
Solved example Throughout this book you will find example problems that have been problems are provided solved for you. Following each solved example, there are often practice problems. The answers to the practice problems are provided so that you can check your work while practicing your problem-
solving skills. Always remember to write out the steps when you are solving problems on your own. If you make a mistake, you will be able to look at your work and figure out where you went wrong. Here is the format for example problems:

| Calculating | A wax candle has a volume of $1,000 \mathrm{ml}$. The candle has a mass of $1.4 \mathrm{~kg}(1,400 \mathrm{~g})$. |  |
| :--- | :--- | :--- |
| density | What is the density of the candle? |  |
|  | 1. Looking for: | You are asked for the density. |
| 2. Given: | You are given the mass and volume. |  |
|  | 3. Relationships: | Density is mass divided by volume: |
| 4. Solution: | density $=(1,400 \mathrm{~g}) \div(1,000 \mathrm{ml})=1.4 \mathrm{~g} / \mathrm{ml}$ |  |

Physics Problem solving is important in all careers. For example, financial problems analysts are expected to look at information about businesses and figure out which companies are succeeding. Doctors collect information about patients and must figure out what is causing pain or an illness. Mechanics gather information about a car and have to figure out how to fix the engine. All these examples use problemsolving skills.

## Step 1

What do you want to find?

## Step 2

## What do you know?

Step 3

## Identify useful relationships

## Step 4

## Solve the problem

Figure 4.4: Follow these steps and you will be able to see a way to the answer most of the time.

## Density of common materials

## Material density is independent of shape

Density is a property of material independent of quantity or shape. For example, a steel nail and a steel cube have different amounts of matter and therefore different masses (Figure 4.5). They also have different volumes. However, if you calculate density by dividing mass by volume, the result is the same for both the nail and the cube.

Solids that are strong, such as steel, typically have high density. High density means there are many atoms per cubic centimeter. Many atoms in a material means many bonds between atoms, and those bonds are what ultimately create strength. There are exceptions, however. Lead is very dense but not very strong because the bonds between lead atoms are relatively weak.

Soft materials typically have lower density

Solids with low density, such as cork or foam, are often used as cushioning material. Low density means there are relatively large spaces between atoms. That means materials may be compressed relatively easily, which is why foam and other low-density substances make good packing materials.

## Liquids tend to be less dense than solids of the same material

The density of a liquid is usually close to solid density, surprisingly enough, but less. For example, the density of solder is $10 \mathrm{~g} / \mathrm{ml}$. The density of liquid solder is 9.5 $\mathrm{g} / \mathrm{ml}$. The liquid density is lower because
 the atoms are not packed as uniformly as they are in the solid. Picture a brand-new box of toy blocks. When you open the box, you see the blocks tightly packed in a repeating pattern, like the atoms in a solid. Now imagine dumping the blocks out of the box, and then trying to pour them back into the original tight packing pattern. The jumbled blocks take up more space, like the atoms in a liquid. Water is an exception to this rule. The density of solid water, or ice, is less than the density of liquid water.

## Steel density

## Steel cube

Volume: 1.0 cm $^{3}$ Mass: 78 g Density: $\mathbf{7 . 8} \mathbf{~ g} / \mathrm{cm}^{3}$


Figure 4.5: The density of a steel nail is the same as the density of a solid cube of steel.


Figure 4.6: The same number (or mass) of blocks arranged in a tight, repeating pattern take up less space than when they are jumbled up.

## Volume

Volume Volume is the amount of space an object takes up. The units used to measure volume depend on whether the object is solid or liquid. The volume of solids is measured in cubic centimeters $\left(\mathrm{cm}^{3}\right)$ or cubic meters $\left(\mathrm{m}^{3}\right)$. The volume of liquids is measured in milliliters ( mL ) or liters ( L ). One cubic centimeter is the same volume as one milliliter.

Measuring the Measuring the volume of liquids is easy. Pour the liquid into a volume of liquids marked container called a graduated cylinder and read the volume. There are two things to keep in mind to measure accurately. First, read the mark at eye level. Second, you will notice that the surface of the liquid forms a curve rather than a straight line (Figure 4.7). This curve is called the meniscus. Read the volume at the center of the meniscus.
Volume of solids You have probably already learned to measure the volume of some solid shapes. The volume of a rectangular solid (a shoebox shape), for example, is found by multiplying length times width times height. The volume of a sphere is $4 / 3 \pi r^{3}$, with $r$ equal to the radius of the sphere.

The You can find the volume of an irregular shape using a technique displacement called displacement. To displace means to "take the place of" or to method "push aside." You can find the volume of an irregularly shaped object by putting it in water and measuring the amount of water displaced.
How you make the measurement

You can use the displacement method to find the volume of an ordinary item like a house key. Fill a 100 -milliliter graduated cylinder with 50 mL of water (Figure 4.7). Gently slide the key into the water. The water level in the container will rise, because the key displaced, or pushed aside, some water. If the level now reads 53.0 mL , you know that the key displaced 3.0 mL of water. The volume of the key, or of any object you measured in this way, is equal to the volume of the water it displaced. The key has a volume of 3.0 milliliters, or 3.0 cubic centimeters $\left(\mathrm{cm}^{3}\right)$.


Figure 4.7: The meniscus of water has a concave shape. Read the mark at the bottom of the curve.


Figure 4.8: The key displaced 3.0 milliliters of water.

## Determining density

Measuring To find the density of a material, you need to know the mass and density volume of a solid sample of the material. You can calculate the density from the formula below.

## DENSITY

$$
\underset{\left(\mathrm{kg} / \mathrm{m}^{3} \text { or } \mathrm{g} / \mathrm{cm}^{3}\right)}{\text { Density }} \boldsymbol{D}=\frac{\boldsymbol{m}}{\boldsymbol{V}} \longleftarrow \text { Mass }(\mathrm{kg} \text { or } \mathrm{g})
$$

Mass is measured with a balance or scale. For irregular objects (Figure 4.10) the displacement method can be used to find the volume.
Calculating For simple shapes you can calculate the volume. The volume of volume spheres, cylinders, and rectangular solids is given in the diagram in Figure 4.9. When calculating volume, all of the units of length involved in the calculation must be the same. For example, if you want volume in cubic centimeters, all of the measurements in the calculation must be in centimeters.


A student measures the mass of five steel nuts to be 96.2 grams. The nuts displace 13 mL of water. Calculate the density of the steel in the nuts.

## Calculate density from mass and volume

| 1. Looking for: | Density |
| :--- | :--- |
| 2. Given: | Mass $(96.2 \mathrm{~g})$ and volume $(13 \mathrm{~mL})$ |
| 3. Relationships: | Density $=$ mass $\div$ volume |
| 4. Solution: | D $=96.2 \mathrm{~g} \div 13 \mathrm{~mL}=7.4 \mathrm{~g} / \mathrm{mL}$ |
| Your turn... |  |

## Your turn...

a. A solid brass block measures $2 \mathrm{~cm} \times 2 \mathrm{~cm} \times 3 \mathrm{~cm}$ and has a mass of 48 g . What is its density? Answer: $4 \mathrm{~g} / \mathrm{cm}^{3}$


Figure 4.9: The volume of some simple geometric shapes.


Figure 4.10: Use the displacement method to find the volume of irregular objects. Use a scale to find the mass.

## Why density varies

Atoms have The density of a material depends on two things. First is the different masses individual mass of each atom or molecule. Solid lead is denser than solid aluminum mostly because a single atom of lead has 7.7 times more mass than a single aluminum atom.

Atoms may be Second, density depends on how tightly the atoms are packed. A "packed" tightly diamond is made of carbon atoms and has a density of $3,500 \mathrm{~kg} / \mathrm{m}^{3}$.
or loosely The carbon atoms in diamonds are closely packed. Paraffin wax is mostly carbon, but the density of paraffin is only $870 \mathrm{~kg} / \mathrm{m}^{3}$. The density of paraffin is low because the carbon atoms are mixed with hydrogen atoms in long molecules that take up a lot of space.

Solving density Density problems usually ask you to find one of the three variables problems (mass, volume, density), given the other two. Figure 4.12 shows three forms of the density equation you can use. Which one you choose depends on what you are asked to find.


Using the density equation

A 4,500-gram cube of titanium is 10 centimeters on each side. Calculate its volume in $\mathrm{cm}^{3}$, and then calculate its density in $\mathrm{g} / \mathrm{cm}^{3}$.

1. Looking for: You are looking for the volume of a solid and its density.
2. Given: You are given the size and mass.
3. Relationships: $\quad V=$ length $x$ width $x$ height $\quad D=m / V$
4. Solution: $\quad V=10 \mathrm{~cm} x 10 \mathrm{~cm} \times 10 \mathrm{~cm}=1,000 \mathrm{~cm}^{3}$

$$
D=4,500 \mathrm{~g} \div 1,000 \mathrm{~cm}^{3}=4.5 \mathrm{~g} / \mathrm{cm}^{3}
$$

## Your turn...

a. Calculate the volume and density of a block that has the dimensions 10 cm x 5 cm x 4 cm and a mass of 400 grams. Answer: $200 \mathrm{~cm}^{3}, 2 \mathrm{~g} / \mathrm{cm}^{3}$
b. A 6 -gram marble put in a graduated cylinder raises the water from 30 mL to 32 mL . Calculate the marble's volume and density. Answer: $2 \mathrm{~cm}^{3}, 3 \mathrm{~g} / \mathrm{cm}^{3}$


Paraffin (density $=870 \mathrm{~kg} / \mathrm{m}^{3}$ )


Figure 4.11: The carbon atoms in diamonds are packed tightly while the carbon atoms in paraffin are not.

| Use. .. | _. if you <br> know $\ldots$ | want to <br> find $\ldots$ |
| :---: | :---: | :---: |
| $D=m \div V$ | mass and <br> volume | density |
| $m=D \times V$ | volume and <br> density | mass |
| $V=m \div D$ | mass and <br> density | volume |

Figure 4.12: Relationships to use when solving density problems.

